CATEGORIFICATIONS FOR LINK INVARIANTS COMING FROM SYMPLECTIC GEOMETRY

CRISTINA PALMER-ANGHEL

I. Khovanov Homology and Symplectic Khovanov Homology

In this series of talks we will discuss about knot invariants and categorifications for knot invariants which come from symplectic geometry. For a given knot invariant, a categorification means a sequence of modules which are knot invariants, whose graded Euler characteristic is precisely the initial invariant.

In 1984, it was discovered the Jones polynomial which is the first quantum invariant for knots and links. Later on, in 2000, Khovanov introduced a categorification for the Jones polynomial, using algebraic and combinatorial techniques. After that, in 2006, Seidel and Smith introduced a sequence of modules which are link invariants, called Symplectic Khovanov Homology, using the geometry of nilpotent slices for the lie algebra sl(m). Then they conjectured that the Khovanov Homology and Symplectic Khovanov Homology are isomorphic. This conjecture was proved in 2015 by Abouzaid and Smith.

In this talk, we will present the definition of the Symplectic Khovanov Homology $KH^*_{\mathrm{sympl}}(L)$. One starts with a link as a closure of a braid $\beta \in B_n$. Firstly, using a nilpotent slice for the algebra $sl(2n)$, it will be constructed a projection map onto the symmetric power of the complex plane, which is a symplectic fibration over the configuration space of $2n$ points. Secondly, using a singular part of the projection map where exactly two points "collide", called singularities of type (A_1), one defines inductively a Lagrangian submanifold $\mathscr{L}$ in the generic fiber. The invariant $KH^*_{\mathrm{sympl}}(L)$ will be defined as a Floer homology in the fiber, between $\mathscr{L}$ and $\beta\mathscr{L}$ (where in the previous we use a parallel transport along β which leads to a diffeomorphism of the fiber).

Paris Diderot University